“ON THE ACTUAL BEHAVIOR OF CREDIT DERIVATIVES BASED ON HOMOGENEOUS REFERENCE PORTFOLIOS”

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We apply an analytical approach to analyze two credit-derivative investments: (i) a synthetic index linked to high-yield corporates; and (ii) an actual transaction based on mortgage bonds.

We show that the conventional approach to analyze these structures (Monte Carlo simulations combined with the Gaussian copula) fails to account for the tri-modal nature of the underlying portfolio defaults distribution, and consequently, risk assessments based on this method give a misguided view of the risk-reward profile of such investments. Furthermore, we show that the benefits of portfolio diversification in the context of credit-risk portfolios are limited in high-correlation scenarios.
The recent financial (subprime) crisis rocked the global markets and shook the foundation of modern economics. Not surprisingly, the crisis has become a favorite research topic among academics, regulators, politicians, investors and journalists as they have tried to identify the reason(s) behind this event. Broadly speaking, we could say that there is (some?) consensus that the crisis was the combined result of inadequate regulation, poor mortgage underwriting standards, serious conflicts of interests at the credit rating agencies, and a deficient understanding of the markets capacity for self-correction [see e.g. Barnett-Hart (2009), U.S. Congress (2010), Basel Committee on Banking Supervision (2008), Acharaya and Richardson (2009)]. Additionally, there is some evidence that lenders who securitized a big proportion of the loans they originated were more likely to relax their covenants and embraced lenient monitoring practices [see e.g. Wang and Xia (2014)].

In terms of its manifestation, a defining characteristic of this crisis was the high number of defaults (and severe downgrades) of many allegedly safe financial products [see e.g. Benmelech and Dlugosz (2009), Faltin-Traeger and Mayer (2011)]. Consequently, we can say that this crisis was also a predictive modeling failure. Therefore, notwithstanding the potential merits of the above-mentioned explanations, focusing on the modeling strategy employed to analyze these products is essential. Only understanding the shortcomings of the models, one can begin to understand what was at the root of this crisis.

The affected products were mostly securitization vehicles or synthetic bonds supported by diversified pools of assets subject to credit risk. Clearly, if one analyzes these structured products making optimistic assumptions regarding the risk associated with the underlying portfolios, the results will be doomed, no matter how appropriate the model. This issue has already been recognized by Heitfield (2009), Rom (2009), and Zapata and Cifuentes (2013).
However, what has yet to be acknowledged, is that if the structure of the model used to estimate the performance of these products is flawed, having a good assessment of the quality of the underlying portfolio (essentially, an input for the modeling process) is useless. Obviously, the combined effect of both, a faulty model plus bad input data, only magnifies the disaster.

In what follows we show, by invoking an analytical solution that is valid in many practical cases, that the most widely-used modeling approach employed to study these structured products (Monte Carlo simulations combined with the Gaussian copula, or MC/GC, for simplicity) was indeed flawed. Specifically, this modeling approach is unable to capture the tri-modal nature that defines the probabilistic behavior of homogeneous pools of credit-risk assets. This shortcoming leads to a significant underestimation of the risk associated with these instruments.

1. The Basic Problem

Consider a homogeneous portfolio of N assets (or bonds, as these terms will be used interchangeably) subject to credit (default) risk. Each bond accounts for the same fraction (1/N) of the entire portfolio. Furthermore, all bonds have the same default probability, p. Let $I_i$ (i=1, …, N) be an index variable (1=default; 0=no default) associated with each bond. We assume that the correlation, $\rho$, between the default behavior of any pair of assets is the same. That is, for all pairs $i \neq j$ (i, j=1, …, N) we have $Correlation (I_i, I_j) = \rho$. Finally, let X be the number of portfolio defaults. Clearly, there are N+1 possible realization scenarios (X= 0, 1, …, N).

The problem consists of estimating the probability of occurrence of each scenario (namely, estimating the probability mass function associated with X). With this information, one
can assess the credit risk profile of the portfolio, or, in the case of a securitization or synthetic transaction, assess the creditworthiness of the different tranches supported by the portfolio.

It might be tempting to believe that a homogenous portfolio is too simplistic to have any practical relevance. But that would be wrong. First, many actual transactions--for example, synthetic collateralized debt obligations--are structured using equally-weighted assets. And second, the widely-traded synthetic Markit indexes are all based on portfolios of equally-weighted assets. Hence, homogenous portfolios constitute a sound basis on which we can explore the merits of different modeling approaches.

2. Analytical Solution

In Pagnoncelli and Cifuentes (2014) the authors presented an analytical expression to calculate the probability of having 0, 1, ..., N defaults in a homogeneous pool of N bonds defined by two parameters, p and ρ, as described in the previous section. Aside from the advantages of having a closed expression--no simulations are required--the correlated binomial (CB) formula provides useful insights that help to understand the behavior of some instruments commonly traded in the market. For completeness, we include the expression here:

\[
\begin{align*}
\delta & = \begin{cases} (1-p)\rho & \text{if } i=0 \\ p\rho & \text{if } i=N \\ 0 & \text{otherwise} \end{cases} \\
\end{align*}
\]

where \( i=0, 1, \ldots, N \); and \( \delta \) is defined as follows:

if \( i=0 \) then \( \delta = (1-p)\rho \); if \( i=N \) then \( \delta = p\rho \); otherwise \( \delta = 0 \).

It follows from Equation (1) that
The most salient feature of this distribution is its tri-modal nature. The probability mass function exhibits a peak around the mean plus two additional peaks, one at \( X=0 \) and the other at \( X=N \). The peak at \( X=0 \) corresponds to a no-default scenario (all bonds perform perfectly). The peak at \( X=N \) corresponds to a "total ruin" scenario (all bonds default). The probabilities associated with these two events increase with the correlation. This fact (as we will see in the examples) has important consequences for the ratings of structured products based on credit-risk portfolios.

2.1 Asymptotic behavior

The asymptotic behavior of total portfolio losses as \( N \) approaches infinity is worth exploring. For simplicity, assume a portfolio with a notional amount of $1. In this case, each default inflicts a $1/N loss to the portfolio (we are assuming no recoveries). Using Equations (2) and (3) it can be shown that the total loss, \( L \), experienced by a portfolio of \( N \) assets is such that

\[
- \]

The common perception that diversification always reduces risk--in this case, represented by the variance-- is challenged as
In short, as N increases the variance approaches a limit, which, of course, cannot be decreased by further diversification. This result implies that correlation renders diversification irrelevant as the number of assets increase. The practical implications of this result are explored in the Examples section.

2.2 Issues with the Gaussian copula

The majority of academic publications, private sector practitioners, and regulators use MC/GC-based approaches as the main tool to analyze structured products [see e.g. Li (2000), Werpachowski (2009), Schonbucher (2003)]. The Gaussian copula, however intuitive and easy-to-understand, has an important shortcoming: it relies on enforcing an asset correlation condition, when in fact, what we really need is to enforce a default correlation condition. The reason for its popularity is that there is plenty of data regarding asset correlation values whereas data on default correlations is sparse at best. Unfortunately, asset correlation is not a good proxy for default correlation, and some undesirable consequences occurred when one tries to model the latter based on the former. This issue has been discussed in detail by Cifuentes and Katsaros (2007).

A second and critical shortcoming of the Gaussian copula (especially in the context of this study) is that it generates only unimodal distributions. Thus, it fails to capture two important features that are heavily influenced by the correlation, namely, the probability of having no defaults (X=0) and the probability of all assets defaulting together (X=N). We claim that by ignoring the tri-modality of the loss distribution, ratings of structured
products based on MC/GC-based approaches can grossly underestimate the risk associated with these instruments.

3. **Examples**

We compare the results obtained with the CB formula and the MC/GS approach using two real transactions.

3.1 **The CDX.NA.HY index**

This example, which is based on the structure of the CDX.NA.HY (North America High Yield) index, is aimed at showing the inadequacy of the MC/GC approach to estimate the credit risk of a pool of assets [Markit (2008)]. Thus, our reference portfolio consists of 100 equally-weighted bonds having all a five-year maturity and all a similar credit-risk profile (in the Ba-B range based on Moody's credit ratings).

According to Moody's data, the observed 5-year default rates for Ba and B corporate bonds are 10.21% and 26.79%, respectively [Moody's Investors Service (2008)]. Based on these data, we assume for our analysis a value of \( p = 18.5\% \) (the average of both figures).

Default correlation values are notoriously hard to estimate. That said, most practitioners usually employ values in the 5%-40% range, depending on their views. There is also some consensus that during financial crisis correlation tends to increase, whereas in "normal" times tends to be low [see e.g. HSBC Global Research (2010)]. Thus, as a first glimpse at the default behavior of this pool we assume \( \rho = 22.5\% \) (the mid-range value).

Figure 1, which displays a continuous plot of the probability mass function of \( X \) (number of portfolio defaults) obtained with the two approaches (the MC/GC and the CB) is disturbing.
The two distributions have strikingly different properties even though they both render identical values for $E(X)$, 18.5 (100 x 0.185) defaults. The distribution estimated with the MC/GC simulation is unimodal whereas the true distribution is tri-modal. The MC/GC-obtained function has a tail that approaches zero asymptotically, suggesting that high defaults scenarios are increasingly unlikely as $X$ grows. The analytical solution --which, we emphasize, captures accurately the default correlation-- indicates that the probability of ruin (100 defaults) is much higher than the value suggested by the Gaussian copula (4.16% versus essentially zero). It also shows that for all practical purposes the probability of the "intermediate" scenarios (roughly, $X$ between 33 and 99) is negligible (of the order of $10^{-7}$).

A critical issue is that the MC/GC underestimates the probability of a catastrophic scenario (100 defaults) significantly. Also relevant (but perhaps less important from a risk perspective) is that the actual probability of having no defaults (18.34%) is much higher than the estimate offered by the MC/GC (1.21%). Furthermore, it should be clear that the behavior displayed in Figure 1 is not an artifact of the correlation value chosen, but an intrinsic feature of the functional form established by Equation (1).

An important consequence of the fact that the probabilities of most scenarios between the second ($X=18$) and third ($X=100$) peaks are almost zero becomes manifest in the context of structured products. Simply put, it might appear that an investor whose tranche is impacted after 80 defaults--for example--is in a safer position than an investor whose tranche is affected after 30 defaults. However, the CB analysis shows that both positions are equally risky. This issue is dealt with, in more detail, in the next example.
In general, the "correct" correlation factor can be very difficult to estimate; thus, prudent analysts perform several analyses using a range of values for \( \rho \). However, one thing is certain—based on a fair amount of empirical evidence, at times of crises correlation increases [see e.g. HSBC Global Research (2010)]. Unfortunately, as Figure 2 shows, portfolio diversification (to be clear, increasing the number of assets in the pool) does not help much when correlation increases. We can see that for a fixed value of \( \rho \) (18.5% in the example), and for values of \( \rho \) higher than 40%, a portfolio of 100 holdings and one of 10 have essentially the same \( \text{Var}[L] \). This is a direct consequence of the result shown in Equation (6). In other words, the benefits of having a highly diversified portfolio vanish in crises situations (when the correlation is high), that is, when it matters most. This result somehow contradicts conventional portfolio theory, at least in the context of credit-risk portfolios. In short: in portfolios whose performance is driven by default events (essentially, binary outcomes) diversification works very differently from more conventional portfolios (stocks, for instance) where performance is driven by smooth price variations. Smooth variations in assets' prices tend to get smoother as the number of assets in the pool increases. Default events—even though their relative impact decreases as the number of assets grow—still retain a "discontinuous" character.

3.2 The Abacus transaction

This synthetic collateralized debt obligation was brought to market in early 2007 and became a cause célèbre due to its poor performance [see e.g. Goldman Sachs (2007)]. A few months after closing the entire reference pool defaulted, the investors were wiped out, and Goldman Sachs—the banker who put together the deal—ended up paying a $550 million fine to the Securities and Exchange Commission [Securities and Exchange Commission (2010)]. The early failure of this transaction somehow contradicted the popular view that deals structured by well-known bankers
did better than deals put together but "lesser known" bankers. In fact, the opposite seems to be true. This point has been discussed extensively in the paper by Griffith et al. (2013).

The deal was a $2 billion synthetic transaction that referenced a portfolio of 90 equally-weighted mortgage-bonds with an (allegedly) average Baa rating and maturities between 3.9 and 4.9 years. Figure 3 shows the schematics of the transaction. The most controversial part of this transaction was in reference to the Class A tranche, which received a Aaa rating, and was purchased by IKB, a German bank. Before the end of 2007 IKB lost its entire investment. Is this something that could have been anticipated?

First, let us examine the portfolio credit-risk profile. To be conservative we assume a maturity of five years (a bit higher than the actual pool average). Allegedly, this pool had originally--at least according to the marketing documents--a Baa average rating. Considering that shortly after the transaction closed the entire pool defaulted, it is reasonable to assume that perhaps the risk profile of the pool was much more speculative than initially thought. With that in mind, we run our simulations using two default probability values, $p=1.94\%$ (which corresponds to a Baa rating) and $p=10.21\%$ (which corresponds to a Ba rating, that is, one notch below). These estimates are based on historic data provided by Moody's Investors Service (2008). We also consider three values for the pool default correlation: 5%, 20% and 40%.

Tables 1 shows, for different default correlations, the expected loss associated with each tranche of the transaction using $p=1.94\%$, estimated by the two methods: the MC/GC and the CB. The table also shows the tranches' ratings based on the corresponding expected loss values [see Yoshizawa (2003)]. Table 2 is similar to Table 1 but displays the results obtained with the Ba-default probability (10.21%).
Several comments are in order.

The most salient observation is that both methods give very different ratings for almost every \((p, \rho)\) pair. Except for the case of the first-loss position, the rating given by the CB approach is always lower than that obtained with the MC/GC method. Interestingly, the CB approach renders no Aaa-ratings under any circumstances. Even more interesting, the CB method gives the A, B, C and D tranches, the same ratings in all cases. The MC/GC, on the contrary, results in a much wider ratings diversity. For example, when \(p=1.94\%\) and \(\rho=40\\%\), all tranches exhibit a different rating according to the MC/GC method.

In essence, the MC/GC approach, from a structured finance viewpoint, gives the impression that one can design six different tranches with six different risk profiles simply by virtue of subordination. However, the CB method proves that this high degree of risk profile variation is indeed an illusion. In fact, in the case of the Abacus transaction only three distinct tranches are possible given the three-peak nature of the pool risk profile. Figure 4 gives away the explanation. For all practical purposes, the default scenarios in the “valley” between the second and third peaks (from 22 up to 89 defaults) are very unlikely to happen: they have an almost-zero probability of occurrence. Thus, in this instance the “protection” offered by subordination is non-existent as the A, B, C and D tranches are all equally risky compared to the super senior. Ironically, they all receive a different coupon, dictated by the misguided notion that their risk profiles are different.

The previous observations indicate that the A tranche, the one bought by IKB, was clearly the worst choice within that same risk profile. Put it differently: tranches A, B, C and D
exhibit all the same risk profile (same expected loss) however, they all offer quite different
coupon payments: the A tranche, the lowest coupon; and the D tranche, the highest coupon.

It is fair to say that the investors--as well as Moody’s--underestimated the effect of the
default correlation as a result of relying on the MC/GC. Tables 1 and 2 show, for a given value
of ρ, not only that the ratings (expected loss) assigned by the CB are lower, but also more
worrisome, that the CB hints that many investment-grade ratings can quickly go into non-
investment territory (for p=10.21% it occurs for correlation values higher than 20%).

It is important to clarify that we are not claiming that our model could have foreseen the
default level actually experienced by the Abacus’ pool. What we claim, however--and this is
relevant from a regulatory viewpoint—is that the transaction was poorly designed since all the
intermediate tranches (A through D) behaved essentially as one “big” tranche. We also claim--
important from a ratings perspective—that the effects of variations in the default correlation
impact the expected loss significantly. Although is difficult to estimate what the correct ρ is, in
practice, investors should be aware of the magnitude of risks they face when markets become
unstable and ρ increases. The CB shows that an apparently safe instrument can quickly
deteriorate below investment-grade level for higher values of ρ.

4. Conclusions

The main conclusion of this study is that risk analyses of pool of assets subject to default risk,
and, more broadly, securitization vehicles based on credit-risk portfolios, should not be analyzed
using the MC/GC approach. This approach fails to grasp the tri-modal distribution nature of the
assets defaults and in general underestimate the risk. More to the point, it underestimates the
(non-negligible) probability that the entire pool could default. And high correlation values only
magnify the significance of this shortcoming. Equally interesting--and somewhat counterintuitive--is the fact that increasing diversification at the pool level, when the correlation is high, has a limited benefit in terms of reducing risk.

Second, regulatory efforts aimed at increasing the subordination level in securitization structures, or, alternatively, establishing minimum sizes for the first-loss position seem misguided. More precisely, increasing the "thickness" of a subordinate tranche might fail to make the tranche above safer, if the improvement in tranche size remains associated with default levels in the low-probability area within the second and third peak of the distribution.

Finally, given the volatility of ratings associated with default correlation changes, the idea of using dispersion measures in conjunction with expected losses metrics should be investigated for rating purposes. This is critical to establish a prudent regulatory framework.
References


Figure 1.

**Probability mass function**

A continuous representation of the probability mass function for a portfolio of 100 assets with $p = 18.5\%$ and $\rho = 22.5\%$ obtained with two different methods: (i) the MC/GC approach; and (ii) the BC approach.
Figure 2.

Var[L] reduction as a function of N.

Value of actual Var[L] (from Equation(5)) divided by its limit as N approaches infinity (from Equation (6)), for three different values of ρ and a given (fixed) value of p (18.5%). The graph shows that as N grows, the benefits of diversification (risk reduction as measured by the variance of L or potential portfolio losses) diminishes. The effect is more salient for higher correlation values.
Figure 3.

Structure of the Abacus transaction.

The left-hand side describes the reference portfolio; the right-hand side depicts the tranching structure.
Figure 4.

Probability mass function (Abacus).

A continuous representation of the probability mass function for a portfolio on 90 assets with \( p = 10.21\% \) and \( \rho = 20\% \) obtained with two different methods: (i) the MC/GC approach; and (ii) the BC approach.
Table 1.

Abacus, expected loss and ratings (p=1.94%)

Abacus transaction: Expected loss and corresponding credit ratings for the different tranches assuming a portfolio default probability, p= 1.94% (Baa rating). The calculations were performed using two different approaches (MC/GC and BC) and three different correlation values.
Table 2.

**Abacus, expected loss and ratings** (p=10.21%)

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Abacus transaction: Expected loss and corresponding credit ratings for the different tranches assuming a portfolio default probability, p=10.21% (Ba rating). The calculations were performed using two different approaches (MC/GC and BC) and three different correlation values.